

Elo Rating system – evolving correction of estimation error

This note is looking at the Elo chess rating system where the rating deviates from underlying strength, and how the situation evolves.

The Elo formula for someone with a rating playing other rated players in an event (in FIDE this is now all results in a month) is

$$R_n = R_o + K(W - W_e)$$

Where

R_n is the new rating,

R_o the pre-event rating

W the total games scores

W_e the expected game score based on R_o

The Elo system was developed assuming no draws; scoring one point for a win zero for a loss.

The formula for W_e – the expected score for a game is:

$$\theta_{ij} = (1 + 10^{-\Delta/400})^{-1}$$

Where $\Delta = R_o - R_j$, R_j being the rating of the opponent.

This is assuming R is a good proxy for λ the true strength of a player.

Using Taylor's series around zero

$$(1 + 10^{-\Delta/400})^{-1} = \left(\frac{1}{2} - \frac{1}{4} \frac{\ln(10)\Delta}{400} \right) \dots \dots \dots$$
$$\approx \frac{1}{2} + (R_o - R_j)/695$$

$$\text{Giving } W_e = n/2 + (nR_o - \sum R_j)/695$$

$$\text{Similarly } E(W) = n/2 + (n\lambda - \sum \lambda_j)/695$$

$$\begin{aligned} \text{So } E(R_n) &= R_o + K[n/2 + (n\lambda - \sum \lambda_j)/695 - (n/2 + (nR_o - \sum R_j)/695)] \\ &= R_o + K/695[(n\lambda - nR_o) + (\sum R_j - \sum \lambda_j)] \end{aligned}$$

Assume $(\sum R_j - \sum \lambda_j)$ is negligible giving

$$\begin{aligned} E(R_n) &= R_o + K/695(n\lambda - nR_o) \\ &= a\lambda + (1-a)R_o \quad \text{where } a = Kn/695 \end{aligned}$$

The point of this note is that it is possible that $a > 1$.